Event-triggering stabilization of complex linear systems with disturbances over digital channels

Mohammad Javad Khojasteh, Mojtaba Hedayatpour, Jorge Cortés, Massimo Franceschetti

Abstract— **As stops and pauses for separating parts of a sentence in language help to convey information, it is also possible to communicate information in communication systems not only by data payload, but also with its timing. We consider an event-triggering strategy that exploits timing information by transmitting in a state-dependent fashion to stabilize a continuous-time, complex, time-invariant, linear system over a digital communication channel with bounded delay and in the presence of bounded system disturbance. For small values of the delay, we show that by exploiting timing information, one can stabilize the system with any positive transmission rate. However, for delay values larger than a critical threshold, the timing information is not enough for stabilization and the sensor needs to increase the transmission rate. Compared to previous work, our results provide a novel encoding-decoding scheme for complex systems, which can be readily applied to diagonalizable multivariate system with complex eigenvalues. Our results are illustrated in numerical simulation of several scenarios.**

I. Introduction

An important aspect of a cyber-physical system [1] is the existence of a finite rate digital communication channel in feedback loop between sensor and controller. Data-rate theorems [2]–[10] quantify the effects of the digital communication channel in the feedback loop on stabilization. Event-triggering control [11]–[13] is also a key component in cyber-physical systems where the objective, in the context of communication, is to minimize the number of transmissions and at the same time ensuring that the control goal is achieved [14]–[17].

While the majority of communication systems transmit information by adjusting the signal amplitude, it is also possible to communicate information by adjusting the transmission time of a symbol [18]–[21]. In a general framework [22] tools from information theory have been utilized to study the fundamental limitation of using timing information for stabilization.

Specifically, it is possible to stabilize a plant using inherent information in the timing of the event. In fact, event-triggering control techniques encode information in the timing in a state-dependent fashion. In this context, a key observation made in [23] states that in absence of the delay in the communication process as well as absence of system disturbances and assuming the controller has knowledge of the triggering strategy, one can stabilize the system with any positive rate of transmission. Our previous work [24] on systems without any disturbance quantifies the information contained in the timing of the triggering events as a function of the delay in the communication channel. For small values of delay in the communication channel, we show that stability can be achieved with any positive transmission rate. However, as the delay increases to values larger than a critical threshold, information implied from the triggering action itself may not stabilize the system and because of that, to ensure stability, the transmission rate must be increased. These results are compared with a time-triggered implementation subject to delay in [14]. In addition to the unknown delay, system disturbances increase the degree of uncertainty in the state estimation process as well. Therefore, to ensure stability, it is crucial to take these effects into account. Our previous work [25] on systems with disturbances derives a sufficient bit rate for stabilization of a scalar linear, real, time-invariant system subjected to bounded disturbance over a digital communication channel with bounded delay.

In this paper, for a system with complex open-loop gain subject to disturbances we derive a sufficient information transmission rate to guarantee stability. More precisely, we design an encoding-decoding scheme that, together with the proposed event-triggering strategy, rules out “Zeno behavior" (an infinite amount of triggering events in a finite-time interval) and ensures that the norm of the state remains bounded as time grows. We show that for small values of delay and using only implicit information, stability can be achieved with an arbitrary positive transmission rate. However, as the delay increases, the information gets old and also corrupted by the system disturbances, therefore higher and higher communication rates are required to ensure stability. In addition, this result sets the basis for the generalization of event-triggered control strategies that meet the bounds on the information transmission rate for the stabilization of vector systems with any real open-loop gain matrix (with complex eigenvalues) under disturbance. Finally, we numerically validate our result in a series of simulations. For reasons of space, proofs are omitted and will appear in full elsewhere.

Notation: Let $\mathbb{R}$, $\mathbb{C}$ denote the set of real and complex numbers, respectively. We let $\log$ and $\ln$ denote the logarithm with bases 2 and e, resp. We denote by $|.|$ and $\|.\|$ the absolute value of a real number and the norm of a complex number resp. Also, any $Q \in \mathbb{C}$ can be written as $Q = \mathrm{Re}(Q) + i\,\mathrm{Im}(Q)$ or $Q = \|Q\| e^{i\phi_Q}$, and for any $y \in \mathbb{R}$

M. J. Khojasteh and M. Franceschetti are with the Department of Electrical and Computer Engineering of University of California, San Diego. M. Hedayatpour is with the Faculty of Engineering & Applied Science, University of Regina, SK, Canada. J. Cortés is with the Department of Mechanical and Aerospace Engineering, University of California, San Diego.{mkhojasteh,massimo,cortes}@ucsd.edu, hedayatm@uregina.ca

we have $\|e^{Qy}\| = e^{\mathrm{Re}(Q)y}$. We denote by $\lfloor x \rfloor$ and $\lceil x \rceil$ the greatest integer less than or equal to x and the smallest integer greater than or equal to x, resp.

II. Problem formulation

We consider a networked control system consisting of a plant, sensor, communication channel and controller, cf. Figure 1. The plant is described by a complex, continuous-

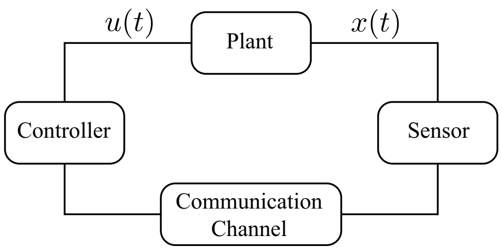

Fig. 1. System model.

time, linear time-invariant model as

$$\dot{x} = Ax(t) + Bu(t) + w(t), \tag{1}$$

where the plant state $x(t)$ and control input $u(t)$ are complex numbers for $t \in [0, \infty)$. Here $w(t) \in \mathbb{C}$ represents a system disturbance, which is upper bounded as

$$\|w(t)\| \leq M,$$

where $M \in \mathbb{R}$ is positive. Here, $A \in \mathbb{C}$, $B \in \mathbb{C}$, and since we are only interested in unstable plants, we also assume $\mathrm{Re}(A) \geq 0$. We consider the next notion of stability.

Definition 1: The plant (1) is practically stable if for any $\|x(0)\| < L$, where $L \in \mathbb{R}$ is nonnegative, there exists an increasing function α of M, with $0 \leq \alpha(0) < \epsilon$ for any $\epsilon > 0$, such that for all $\Psi > \alpha(M)$, there exists T such that, $\|x(t)\| \leq \Psi$ for all $t \geq T$.

The sequence of triggering times where the sensor transmits a data payload packet of length $g(t_s^k)$ bits is represented by $\{t_s^k\}_{k\in\mathbb{N}}$. This packet is delivered to the controller without error and entirely but with unknown upper bounded delay described as follows. Let $\{t_c^k\}_{k\in\mathbb{N}}$ be the sequence of times where the controller receives the packet transmitted at time $\{t_s^k\}_{k\in\mathbb{N}}$ and decodes it, then we have

$$\Delta_k = t_c^k - t_s^k \leq \gamma, \tag{2}$$

with Δ_k being the k^{th} *communication delay*, and γ be a non-negative real number. Also, for all $k \geq 1$, we define the k^{th} *triggering interval* as

$$\Delta'_k = t_s^{k+1} - t_s^k.$$

From this point on, when referring to a generic triggering or reception time, for convenience we skip the super-script k in t_r^k and t_c^k.

We denote by $b_c(t)$ the number of bits that controller received until time t, and we define *information access rate* as

$$R_c = \limsup_{t\to\infty} \frac{b_c(t)}{t}.$$

The number of bits transmitted by the sensor, up to time t is represented by $b_s(t)$ and the *information transmission rate* is defined as follows:

$$R_s = \limsup_{t\to\infty} \frac{b_s(t)}{t}.$$

As the sensor transmits $g(t_s)$ bits of information at each triggering interval, we can write

$$R_s = \limsup_{N\to\infty} \frac{\sum_{k=1}^{N} g(t_s^k)}{\sum_{k=1}^{N} \Delta'_k}.$$

In order to establish a common ground to compare with the information transmission rate later, we state the generalization of the classical data-rate theorem for the complex plant (1).

Theorem 1: Consider the plant-sensor-channel-controller model with plant dynamics (1). If $x(t)$ remains bounded as t approaches infinity then

$$R_c \geq \frac{2\,\mathrm{Re}(A)}{\ln 2}.$$

We also represent the estimated state at the controller by $\hat{x}$ which evolves at each inter-reception interval as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t), \quad t \in [t_c^k, t_c^{k+1}], \tag{3}$$

starting from $\hat{x}(t_c^{k+})$ with $\hat{x}(0) = \hat{x}_0$. We assume that the sensor has causal knowledge of the time of each control action to make sure it can compute $\hat{x}(t)$ for all time t. This is equivalent to establishing an instantaneous acknowledgment link between the sensor and actuator using the control input, as in [26], [27]. For instance, this can be implemented in our system by monitoring the actuator output, which changes at each reception time. Assuming the sensor has only access to the plant state, a narrowband signal can be used in the control input for exciting a specific frequency of the state to inform the sensor about the exact time of each control action by the actuator. That being said, we define the *state estimation error* as

$$z(t) = x(t) - \hat{x}(t),$$

where $z(0) = x(0) - \hat{x}_0$. We rely on this error to determine when a triggering event occurs in our controller design, as explained next.

III. Event-triggered control design

In this section we propose our event-triggering design, and utilize it to find a sufficient condition on the information transmission rate.

Here we describe a class of event-triggered control strategies to determine the sequence of triggering times so that the plant (1) is practically stable. A triggering occurs at t_s^{k+1} if

$$\|z(t_s^{k+1})\| = J, \tag{4}$$

provided $t_c^k \leq t_s^{k+1}$ for natural number k and $t_s^1 \geq 0$. At each triggering time t_s the packet $p(t_s)$ of size $g(t_s)$ is transmitted from the sensor to the controller. The packet $p(t_s)$ consists of the quantized version of phase of $z(t_s)$, denoted by $\phi_{q(z(t_s))}$, and a quantized version of the triggering time t_s. By (4) we have

$$z(t_s) = Je^{i\phi_{z(t_s)}},$$

hence using the packet, a quantized version of $z(t_s)$, denoted by $q(z(t_s))$, at the controller is constructed as follows

$$q\left(z(t_s)\right) = Je^{i\phi_{q(z(t_s))}}.$$

Furthermore, utilizing the bound (2) and the received packet, the controller constructs a quantized version of t_s, denoted by $q(t_s)$. Consequently, at controller $z(t_c)$ can be estimated as follows

$$\bar{z}(t_c) = e^{A(t_c - q(t_s))} q\left(z(t_s)\right). \tag{5}$$

Based on this, one can formulate a procedure, that we term *jump strategy*, to update the estimate of the state maintained by the controller.

$$\hat{x}(t_c^+) = \bar{z}(t_c) + \hat{x}(t_c). \tag{6}$$

Then we can write

$$\|z(t_c^+)\| = \|x(t_c) - \hat{x}(t_c^+)\| = \|z(t_c) - \bar{z}(t_c)\|.$$

At the sensor, the packet size $g(t_s)$ is chosen to be large enough such that the following equation for all $t_c \in [t_s, t_s + \gamma]$ is satisfied.

$$\|z(t_c^+)\| = \|z(t_c) - \bar{z}(t_c)\| \leq \rho_0 J. \tag{7}$$

where $0 < \rho_0 < 1$ and is considered as a design parameter. Under this design, the frequency with which transmission events are triggered is captured by the triggering rate

$$R_{tr} = \limsup_{N \to \infty} \frac{N}{\sum_{k=1}^{N} \Delta'_k}. \tag{8}$$

A typical realization of $z(t)$ under the proposed event-triggering strategy before and after one triggering is represented in Figure 2.

A. Sufficient condition on the transmission rate

In this section, we derive sufficient condition on the information transmission rate to ensure the plant (1) is practically stable. In other words, we design an encoding-decoding scheme that, together with our event-triggering policy, rules out "Zeno behavior" (an infinite amount of triggering events in a finite-time interval) and guarantees the plant (1) is practically stable. The results presented here can be readily applied to multivariate linear plants with disturbance and diagonalizable open-loop gain matrix (with complex eigenvalues).

B. Design of quantization policy

We devote the first λ bits of the packet $p(t_s)$ for quantizing the phase of $z(t_s)$. The proposed encoding algorithm,

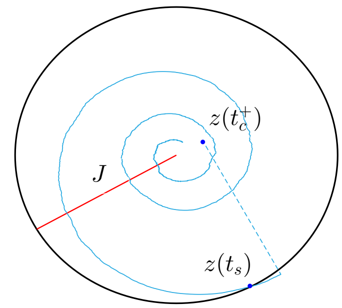

Fig. 2. The blue graph represents evolution of the state estimation error in time before and after a triggering event. The trajectory starts with an initial state inside a circle of radius J, and continues in a disturbed fashion along a spiral trajectory (due to the imaginary part of A) until it hits the triggering threshold radius J, then it jumps back inside the circle after the update according to (5). During inter-reception time intervals we have $\dot{z}(t) = Az(t) + w(t)$, and the overshoot from the circle observed in the trajectory is due to the unknown delay in the communication channel. In this example, $A = 0.3{+}2i$, $B = 0.2$, $u(t) = -8\hat{x}(t)$, $M = 0.2$, $\gamma = 0.09$ sec, $\rho_0 = 0.9$ and $J = 0.147$.

uniformly quantizes the circle to 2^λ pieces of $2\pi/2^\lambda$ radians. After reception, the decoder finds the right phase quantization cell and selects its center point as $\phi_{q(z(t_s))}$. By letting

$$\omega = \phi_{z(t_s)} - \phi_{q(z(t_s))},$$

as depicted in Figure 3, we deduce:

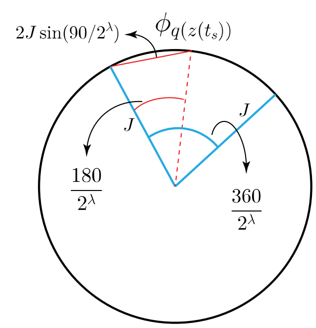

Fig. 3. Estimation of phase angle after triggering event and transmission of λ bits.

$$|\omega| \leq \frac{\pi}{2^\lambda}.$$

Furthermore, we use the encoding scheme proposed in [24] to append a quantized version of triggering time t_s of length $g(t_s) - \lambda$ to the packet $p(t_s)$. As shown inf Figure 4, to determine the time interval of the triggering event, we break the positive time line into intervals of length $b\gamma$. At the controller, after receiving the packet at t_c, t_s could fall anywhere between $t_c - \gamma$ and t_c. Also, after breaking the positive time line into intervals, t_s falls into $[jb\gamma, (j+1)b\gamma]$ or $[(j+1)b\gamma, (j+2)b\gamma]$ with j being a natural number. Therefore, we use the $(\lambda{+}1)^{th}$ bit of the packet to determine the correct interval of t_s. This bit is zero if the nearest integer less than or equal to the beginning number of the interval is an even number and is 1 otherwise. This can be written mathematically as $p(t_s)[\lambda+1] = \mathrm{mod}\big(\lfloor \frac{t_s}{b\gamma} \rfloor, 2\big)$. For the

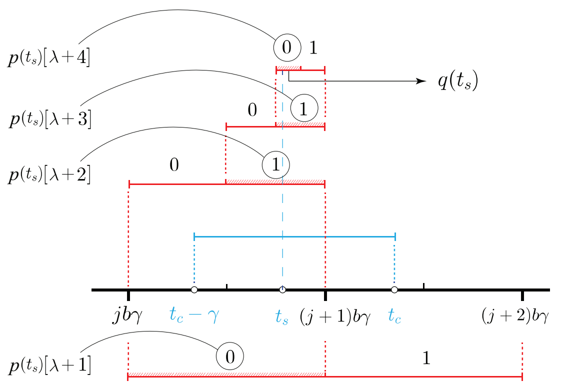

Fig. 4. Quantization of the triggering time t_s. The packet $p(t_s)$ of length $\lambda+4$ can be generated and sent to the controller. After reception and decoding the controller choose the center of the smallest sub-interval as its estimation of t_s, denoted by $q(t_s)$.

remaining bits of the packet, the encoder breaks the interval containing t_s into $2^{g(t_s)-\lambda-1}$ equal sub-intervals. Once the packet is complete, it is transmitted to the controller where it is decoded and the center point of the smallest sub-interval is selected as the best estimate of t_s. Therefore, we have

$$|t_s - q(t_s)| \le \frac{b\gamma}{2^{g(t_s)-\lambda}}.$$

C. Sufficient information transmission rate

Here, we rely on the quantization policy designed above to establish a sufficient bound on the information transmission rate that ensures that the plant is practically stable. We start by showing that we can achieve (7) with the quantization policy.

Theorem 2: Consider the plant-sensor-channel-controller model with plant dynamics (1), estimator dynamics (3), triggering strategy (4), and jump strategy (6). If the controller has enough information about $x(0)$ such that state estimation error satisfies $\|z(0)\| < J$, then the quantization policy designed above achieves (7) for all $k \in \mathbb{N}$ with packet size lower bounded as

$$g(t_s) \ge g' := \tag{9}$$

$$\max\left\{0, \lambda + \log \frac{\mathrm{Re}(A)b\gamma}{\ln\left(\frac{1+e^{-\mathrm{Re}(A)\gamma}\left(\rho_0 - \frac{M}{\mathrm{Re}(A)J}\left(e^{\mathrm{Re}(A)\gamma}-1\right)\right)}{2\sin(\pi/2^{\lambda+1})+1+\sqrt{2\zeta}}\right)}\right\},$$

provided, $\cos\Big(\mathrm{Im}(A)\big(t_s - q(t_s)\big)\Big) = 1-\zeta$, $b > 1$,

$$\rho_0 \ge \tag{10a}$$
$$\frac{M}{\mathrm{Re}(A)J}\left(e^{\mathrm{Re}(A)\gamma}-1\right) + e^{\mathrm{Re}(A)\gamma}\left(2\sin(\pi/2^{\lambda+1}) + \sqrt{2\zeta}\right),$$

$$J \ge \frac{M}{\mathrm{Re}(A)\chi}\left(e^{\mathrm{Re}(A)\gamma}-1\right), \tag{10b}$$

$$\sqrt{2\zeta}e^{\mathrm{Re}(A)\gamma} \le \chi', \tag{10c}$$

and

$$\lambda > \log\left(\frac{\pi}{\arcsin\left(\frac{1-\chi-\chi'}{2e^{\mathrm{Re}(A)\gamma}}\right)}\right) - 1, \tag{10d}$$

where $0 < \chi + \chi' < 1$.

Next we show that, using our encoding-decoding scheme, if the sensor has a causal knowledge of the delay in the communication channel, it can calculate the state estimation for all time.

Proposition 1: Consider the plant-sensor-channel-controller model with plant dynamics (1), estimator dynamics (3), triggering strategy (4), and jump strategy (6). Using (5) and the quantization policy depicted in Figure 3 and 4, if the sensor has causal knowledge of the delay in the communication channel, then it can calculate $\hat{x}(t)$ for all time t.

We continue by showing that our event-triggered scheme does not suffer from Zeno behavior.

Lemma 1: Consider the plant-sensor-channel-controller model with plant dynamics (1), estimator dynamics (3), triggering strategy (4), and jump strategy (6). If the packet size satisfies (7) for all $k \in \mathbb{N}$, then for all $k \in \mathbb{N}$, we have

$$t_s^{k+1} - t_s^k \ge \frac{1}{\mathrm{Re}(A)} \ln\left(\frac{J + \frac{M}{\mathrm{Re}(A)}}{\rho_0 J + \frac{M}{\mathrm{Re}(A)}}\right).$$

By Lemma 1 we deduce the triggering rate (8) is upper bounded as follows

$$R_{tr} \le \frac{\mathrm{Re}(A)}{\ln\left(\frac{J+\frac{M}{\mathrm{Re}(A)}}{\rho_0 J + \frac{M}{\mathrm{Re}(A)}}\right)}$$

which is valid for all realization of the plant disturbance, initial conditions, and delay. Combining this bound with Theorem 2, we obtain the following result.

Corollary 1: Under the assumption of Theorem 2, there exists a quantization policy that achieves (7) for all $k \in \mathbb{N}$ and for all delays and plant disturbance realizations with an information transmission rate

$$R_s \ge \frac{\mathrm{Re}(A)}{\ln\left(\frac{J+\frac{M}{\mathrm{Re}(A)}}{\rho_0 J + \frac{M}{\mathrm{Re}(A)}}\right)} g', \tag{11}$$

where g' is defined in (9), provided $\cos\Big(\mathrm{Im}(A)\big(t_s - q(t_s)\big)\Big) = 1-\zeta$, $b > 1$, (10a), (10b), (10c), and (10d).

Figure 5 shows the sufficient information transmission rate (11) as a function of the channel delay upper bound γ. One can observe that for small value of the delay, the sufficient information transmission rate is smaller than the rate required by the extension of the data-rate result in Theorem 1. As the delay upper bounded increases, the sufficient information transmission rate increases accordingly.

As a by-product of the above discussion, we can guarantee (1) is practically stable, as stated in the following result.

Theorem 3: Under the assumption of Theorem 2, when the pair (A, B) is stabilizable, using the control rule $u(t) = -K\hat{x}(t)$ the system (1) is practically stable provided that

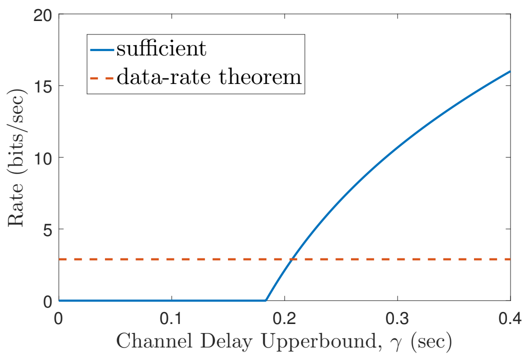

Fig. 5. Sufficient transmission rate (11) as functions of channel delay upper bound γ. We assume $A = 1 + i$, $B = 0.5$, $M = 0.1$, $\rho_0 = 0.9$ and $b = 1.0001$. Also $\lambda = \log\left(\pi/2 \arcsin(\frac{7}{8}) e^{\mathrm{Re}(A)\gamma}\right)$ and $J = \frac{8M}{\mathrm{Re}(A)}\left(e^{\mathrm{Re}(A)\gamma} - 1\right) + 0.002$. In this case, the rate dictated by data-rate theorem (Theorem 1) is $2\,\mathrm{Re}(A)/\ln 2 = 2.885$.

information transmission rate is lower bounded by (11) and the real part of $A - BK$ is negative.

Remark 1: Our previous work [24] extends the results on event-triggered control for plants without disturbance from the scalar to the vector case, but is limited to plants with real eigenvalues of the open-loop gain matrix. Our discussion for complex plants here sets the basis for generalizing this result to plants subject to disturbance and for any real open-loop gain matrix (not necessarily with real eigenvalues). •

IV. Simulation Results

This section presents simulation results for stabilization of a complex linear time-invariant plant with disturbances. Although the results are stated for continuous systems, all the simulations are done in a digital environment by sampling the continuous system at a high frequency (such that the resulting approximate system is close to the continuous system). We choose the sampling time $\delta' = 0.002$ seconds. The minimum upper bound for the channel delay is equal to two sampling times.

We consider the state and state estimation as defined in (1) and (3) where $A = 2+0.5i$, $B = 0.5$, and the control input is chosen as $u(t) = -8\hat{x}(t)$. Using (10b), the triggering radius J (cf. (4)) can be found as follows:

$$J = \frac{8M(e^{\mathrm{Re}(A)\gamma} - 1)}{\mathrm{Re}(A)} + \delta',$$

Also, to quantize the phase, using (10d) we calculate λ as follows:

$$\lambda = \left\lceil \log\left(\frac{\pi}{\arcsin\left(\frac{7/8}{2e^{\mathrm{Re}(A)\gamma}}\right)}\right) \right\rceil$$

We carry out a set of three different simulations. In *simulation (a)*, we assume the plant disturbance is zero and the channel delay is upper bounded by two sampling times $2\delta'$. In *simulation (b)*, we assume the plant disturbance is upper bounded by $M = 0.1$ and the channel delay is upper bounded by two sampling times. Finally, for *simulation (c)*, we assume the plant disturbance is upper bounded by $M = 2$ and the channel delay is upper bounded by $\gamma = 1.2$ seconds.

Simulation results are presented in Figure 6, where the first row represents norm of the error $\|z(t)\|$, and triggering radius J, the second row represents the evolution of $\phi_{x(t)}$ and the third row represents the evolution of $\|x(t)\|$ in time. In the third column, despite having large delays and large disturbances, the controller is able to stabilize the plant. As we can see in the plot, the estimate of the state at the controller tracks the norm and phase of the state. In the first row, sudden changes in the norm of the state estimation error represent reception of the transmitted packet at the controller.

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V. Conclusions

We have presented an event-triggered control scheme for stabilization of a continuous-time, complex, time-invariant, linear system over a digital communication channel with bounded delay and in the presence of bounded system disturbances. We have tested the proposed control scheme on an unstable linear system with complex open-loop gain and validated the results in simulation. Future work will study the identification of necessary conditions on the transmission rate in complex systems, developing an event-triggering design for vector systems with real and complex eigenvalues based on the complex system design, and testing of the proposed control strategies on practical scenarios.

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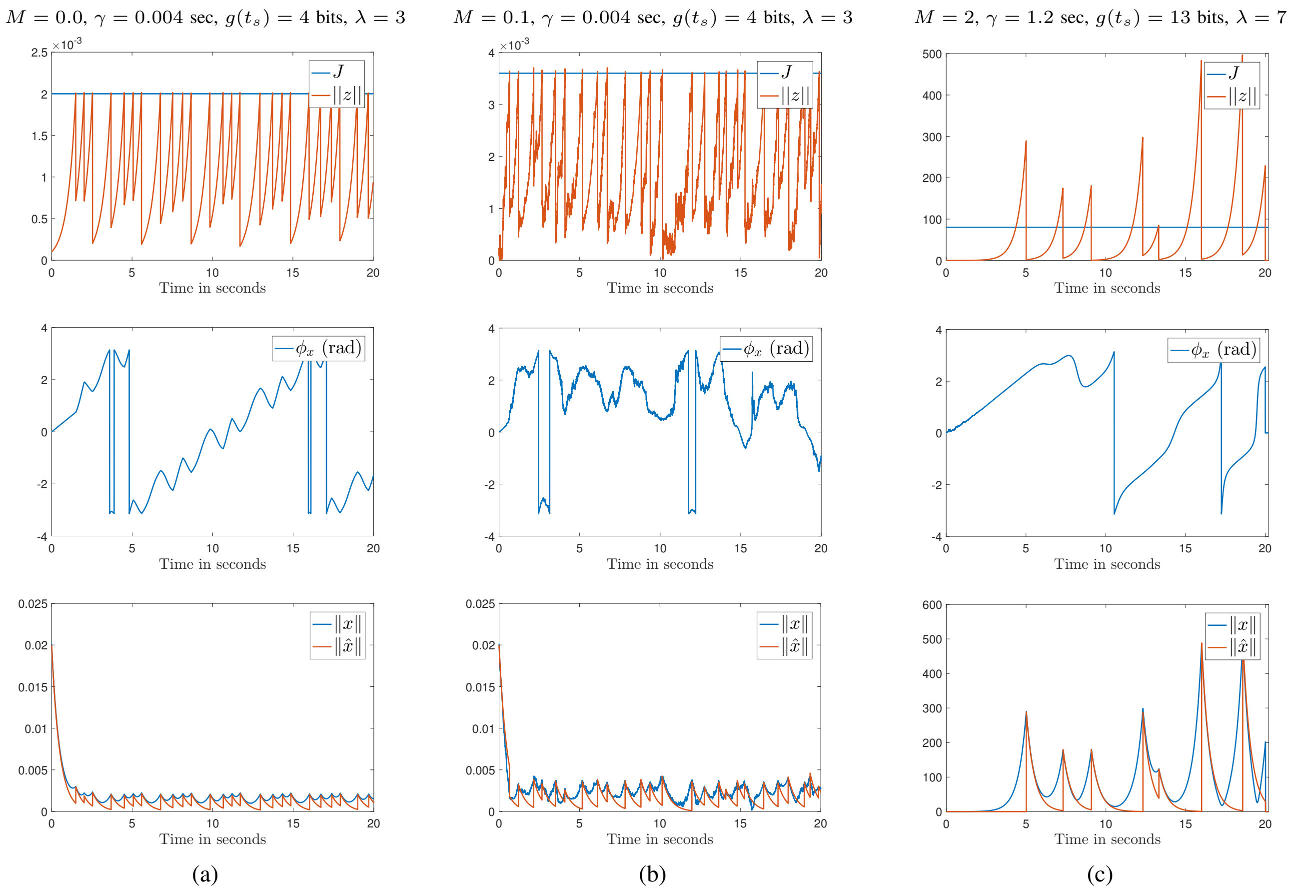

Fig. 6. The first row represents norm of state estimation error. The second row represents variations of phase angle of the complex state x. Finally, and the last row represents evolution of the real component of x and $\hat{x}$ in time. In these simulations $A = 2.5 + 0.5i$, $B = 0.5$, $\rho_0 = 0.9$, $b = 1.0001$, $x_0 = 0.2001$, $\hat{x}_0 = 0.2000$, $\delta' = 0.002$ second.

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