A Unified Approach to Configuration-based Dynamic Analysis of Quadcopters for Optimal Stability

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Outline

Introduction

- Unmanned Aerial Vehicles (UAVs) have been one of the most popular research topics in recent years with lots of applications in:
    - Delivery
    - Agriculture
    - Helping police
    - Wildlife monitoring
    - Military
    - First responders
    - and much more...

Figure 1: Applications of UAVs. Images from [1]-[3]

Introduction: motivation

- So many different configurations exist
- Finding the best configuration to achieve maximum stability/maneuverability
- Lack of a complete mathematical model to represent salient aerodynamic effects in different configurations

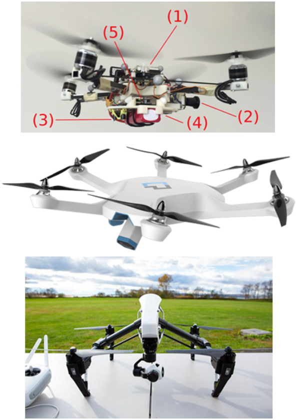


Figure 2: Quadcopter in different configurations [4]-[6]

Introduction: Focus of this paper

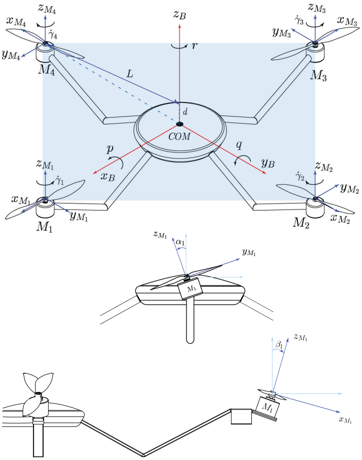


Figure 3: A quadcopter with dihedral and twist angles.

- Complete mathematical modeling considering all forces & moments in the system
- The effects of dihedral and twist angles on stability/maneuverability
- Effects of the location of COM on stability

Mathematical Modeling

Rotational motion: as expressed in the body frame

$$\boldsymbol{\tau} = I^B\dot{\boldsymbol{\omega}}_{B,I} + {}^B\boldsymbol{\omega}_{B,I} \times (I^B\boldsymbol{\omega}_{B,I} + \sum_{i=1}^{4} I^p\boldsymbol{\omega}^{p_i}) \tag{1}$$

$${}^{M_i}\mathbf{F}_{P_i} = [0, 0, k_f\dot{\gamma}_i^2]^T$$

$${}^{M_i}\boldsymbol{\tau}_{P_i} = (-1)^{i+1}k_t{}^{M_i}\mathbf{F}_{P_i}$$

$$\boldsymbol{\tau} = \textstyle\sum_{i=1}^{4}({}^B\mathbf{O}_{M_i} \times {}^B\mathbf{R}_{M_i}{}^{M_i}\mathbf{F}_{P_i} + {}^B\mathbf{R}_{M_i}{}^{M_i}\boldsymbol{\tau}_{P_i})$$

Translational motion:

$$m\ddot{\mathbf{s}} = {}^I\mathbf{R}_B \sum_{i=1}^{4}({}^BR_{M_i}{}^{M_i}\mathbf{F}_{P_i}) + m\mathbf{g} + \boldsymbol{f}_d \tag{2}$$

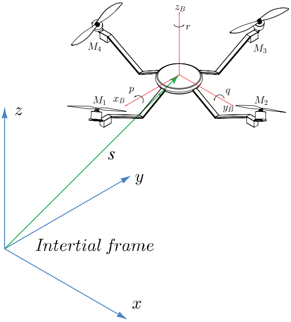


Figure 4: Inertial frame is shown in blue and body frame is shown in red.

Effects of Dihedral & Twist Angles

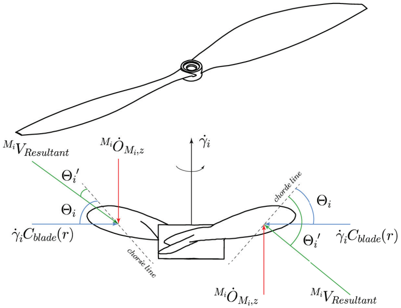


Figure 5: Dihedral Effect - On top is a propeller and on bottom is a front view of it. In the left, is the case when moving the motor up and in the right, is the case when moving the motor down.

- Any air flow with positive (negative) z-component velocity in frame M_i increases (decreases) the AOA which increases (decreases) thrust force.

$$\frac{\Delta C_l}{\Delta \Theta_i} = \sigma$$

$$^{M_i}\dot{\mathbf{O}}_{M_i,I} = {}^{M_i}[\dot{O}_{M_i,x}, \dot{O}_{M_i,y}, \dot{O}_{M_i,z}]^T$$

$$\Delta\Theta_i = \Theta_i - \Theta_i' = arctan(\frac{\dot{O}_{M_i,z}}{\dot{\gamma}_i r})$$

$$^{M_i}\Delta\mathbf{F}_{P_i} = [0, 0, -\frac{1}{4}c\sigma\rho\dot{O}_{M_i,z}|\dot{\gamma}_i|C_{blade}^2]^T$$

Effects of Dihedral & Twist Angles

$$
\begin{aligned}
{}^{M_i}\Delta\mathbf{F}_{P_i,roll} &= [0, 0, -\zeta_{roll}\dot{O}_{M_i,z}]^T \\
{}^{M_i}\Delta\mathbf{F}_{P_i,pitch} &= [0, 0, -\zeta_{pitch}\dot{O}_{M_i,z}]^T \\
{}^{M_i}\Delta\mathbf{F}_{P_i,yaw} &= [0, 0, -\zeta_{yaw}\dot{O}_{M_i,z}]^T \\
{}^{M_i}\Delta\mathbf{F}_{P_i} &= {}^{M_i}\Delta\mathbf{F}_{P_i,roll} + {}^{M_i}\Delta\mathbf{F}_{P_i,pitch} + {}^{M_i}\Delta\mathbf{F}_{P_i,yaw}
\end{aligned}
$$

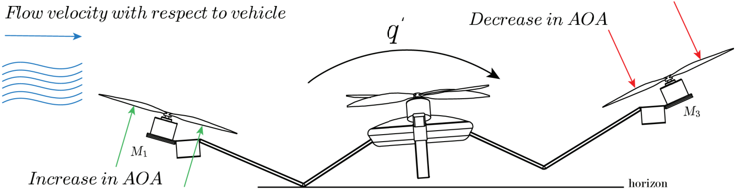


Figure 6: Dihedral effect in 2D motion of quadcopter. The quadcopter is pitching down and moving to the left. Dihedral effect generates the moment q' and acts like damping in the system.

Stability Analysis: Yaw Motion

$$\tau_{yaw} = I_{zz}\dot{r}$$

$$u = \dot{\gamma_1}^2 - \dot{\gamma_2}^2 + \dot{\gamma_3}^2 - \dot{\gamma_4}^2$$

$$\dot{r} = \frac{(k_t k_f c_a - k_f L s_a)}{I_{zz}} u$$

$$C_1 = \frac{(k_t k_f c_a - k_f L s_a)}{I_{zz}}$$

$$\frac{r(s)}{u(s)} = \frac{C_1}{s} \tag{3}$$

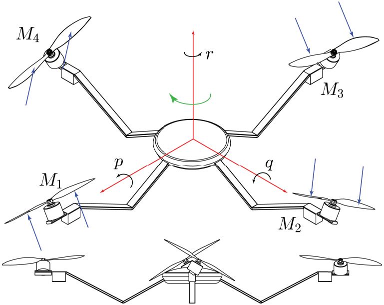


Figure 7: Quadcopter having only twist angles $\alpha_{1,3} > 0$ and $\alpha_{2,4} < 0$. The vehicle is going through pure yaw motion r and dihedral effect generates a counteracting yaw motion that damps yaw motion.

Stability Analysis: Yaw Motion

Adding the moments due to twist angle:

$$^{B}\mathbf{v}_{P_i} = {}^{B}\mathbf{O}_{M_i} \times [0, 0, r]^T$$

$$^{M_i}\mathbf{\Delta F}_{P_i,twist} = -\zeta_{yaw} {}^{B}\mathbf{R}_{M_i}^{T} {}^{B}\mathbf{v}_{P_i}$$

$$\boldsymbol{\tau}_{twist} = \textstyle\sum_{i=0}^{4} {}^{B}\mathbf{O}_{M_i} \times {}^{B}\mathbf{R}_{M_i} {}^{M_i}\mathbf{\Delta F}_{P_i,twist}$$

$$\boldsymbol{\tau}_{twist} = [0, 0, -4\zeta_{yaw} L^2 s_a^2 r]^T$$

$$\tau_{yaw} + \tau_{twist,yaw} = I_{zz}\dot{r}$$

$$\frac{r(s)}{u(s)} = \frac{C_1}{s + \frac{\zeta'_{yaw}}{I_{zz}}} \;, \quad \zeta'_{yaw} = 4\zeta_{yaw} L^2 s_a^2 > 0 \qquad (4)$$

Analytically, it is shown that this configuration leads to a more stable motion.

Stability Analysis: Simulation Results

Numerical results are presented for a vehicle of mass 0.5 kg, $I_{zz} = 5.5 \times 10^{-3}$ kg.m^2, L=0.2 m, $k_f = 6.41 \times 10^{-6}$ N.s^2/rad^2, $k_t = 1.62 \times 10^{-2}$ m, $\gamma = 2.8 \times 10^{-3}$ N.m.s/rad and $g = 9.81$ m/s^2 and $\alpha = 0.3$ rad.

$$\tau_{yaw} + \tau_{twist,yaw} + \tau_{drag} = I_{zz}\dot{r} \tag{5}$$

$$\tau_{drag} = -\gamma r = -0.0028r$$

$$\boldsymbol{\tau}_{twist} = -4\zeta_{yaw}L^2 s_a^2 r = -0.0011r$$

Note that the moment due to twist angle is almost as significant as the moment due to drag in rotational motion.

Stability Analysis: Simulation Results

Assume the vehicle is hovering and we have a disturbance in yaw motion. The response of the system in yaw motion can be simulated as follows. (the same can be done for pitch and roll motions)

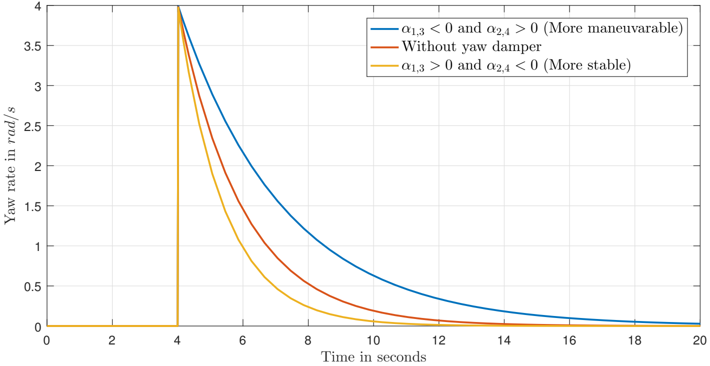
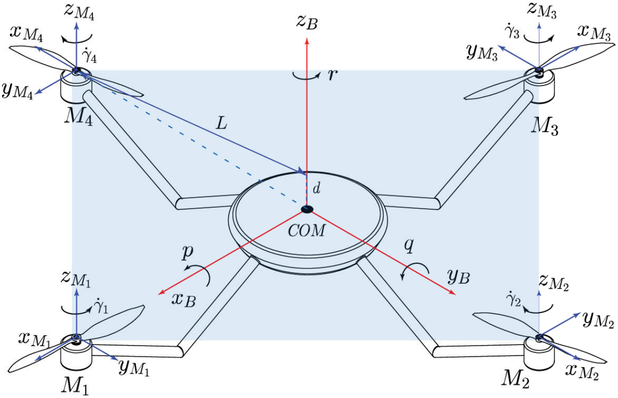


Figure 8: Simulation results - response of the system to disturbance in yaw motion.

Effecs of Location of COM on Stability

The effect of location of center of mass is hidden in the value of ζ' in roll and pitch motion. However, these effects are negligible compared to the effects of dihedral and twist angles.



Configurations' Ranking

The following list, ranks all configurations from the most stable to the most maneuverable (for simplicity, we assume that d is positive for all configurations):

1. $\beta_i < 0$, $\alpha_{1,3} > 0$ and $\alpha_{2,4} < 0$ (most stable)
2. $\beta_i < 0$, $\alpha_i = 0$
3. $\beta_i = 0$, $\alpha_{1,3} > 0$ and $\alpha_{2,4} < 0$
4. $\beta_i = 0$, $\alpha_i = 0$ (regular quadcopter with no tilting angles)
5. $\beta_i = 0$, $\alpha_{1,3} < 0$ and $\alpha_{2,4} > 0$
6. $\beta_i > 0$, $\alpha_{1,3} < 0$ and $\alpha_{2,4} > 0$ (most maneuverable)

Conclusions

- A complete mathematical model for a quadcopter with dihedral and twist angles is presented.
- The effects of dihedral and twist angles on stability are presented analytically.
- Six different configurations based on stability and maneuverability for quadcopters are presented.
- The most stable configuration is perfect for applications where precise hovering is required.
- The most maneuverable configuration is perfect for applications where agility is required.

Future Works

- A reconfigurable system can be designed in a way to transform from the most stable system to the most maneuverable system in the respective situation and vice versa.
- Two different optimization problems can be defined: 1) optimizing the angles for the most stable configuration; and 2) optimizing the angles for the most maneuverable configuration.
- Finally, verifying the results of this paper using experiments can be another topic for future work.

Thank You!

References

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